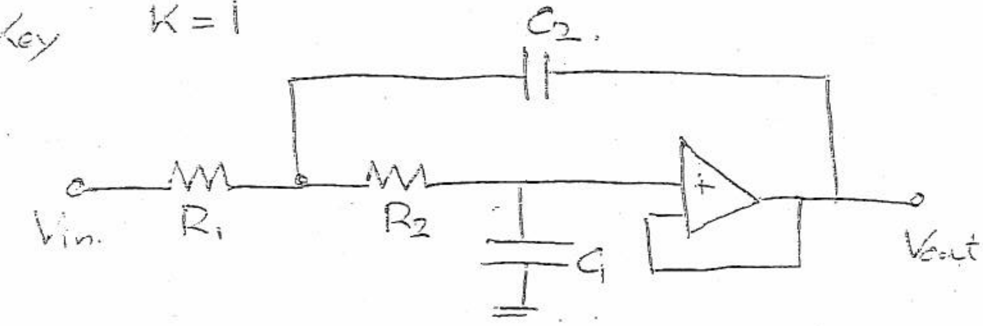
1. $R_1 = 25\ k\Omega$

$C_2 = 50\ pF$

$Q_{max} = 4$

In this Sallen Key $K = 1$



$Y_1 = \frac{1}{R_1}$

$Y_2 = sC_2$

$Y_3 = \frac{1}{R_2}$

$Y_4 = \cancel{\frac{1}{C_1}}\ sC_1$

Transfer function $\frac{V_o}{V_{in}} = \frac{KY_1Y_3}{(Y_1+Y_2)(Y_3+Y_4)+Y_3Y_4-KY_2Y_3}$

Substituting

$$\frac{V_o}{V_{in}} = \frac{\frac{1}{R_1R_2}}{\left(\frac{1}{R_1}+sC_2\right)\left(\frac{1}{R_2}+sC_1\right)+\frac{1}{R_2}\cdot sC_1-\frac{sC_2}{R_2}}$$

Multiplying by R_1R_2

$$\frac{V_o}{V_{in}} = \frac{1}{(1+sC_2R_1)(1+sC_1R_2)+sC_1R_1-sC_2R_1}$$

$$= \frac{1}{1+\cancel{sC_2R_1}+sC_1R_2+s^2C_1C_2R_1R_2+sC_1R_1-\cancel{sC_2R_1}}$$

$$= \frac{1}{s^2C_1C_2R_1R_2+sC_1(R_1+R_2)+1}$$

Dividing everything by $C_1C_2R_1R_2$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{1/C_1C_2R_1R_2}{s^2 + \frac{s\cancel{C_1}(R_1+R_2)}{\cancel{C_1}C_2R_1R_2} + \frac{1}{C_1C_2R_1R_2}}$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{1/C_1C_2R_1R_2}{s^2 + s\cdot\frac{(R_1+R_2)}{C_2\cdot R_1R_2} + \frac{1}{C_1C_2R_1R_2}}$$

Comparing this with

$$\frac{V_o(s)}{V_{in}(s)} = \frac{\omega_o^2}{s^2 + s\,\frac{\omega_o}{Q} + \omega_o^2}$$

$$\omega_o = \frac{1}{\sqrt{C_1C_2R_1R_2}}$$

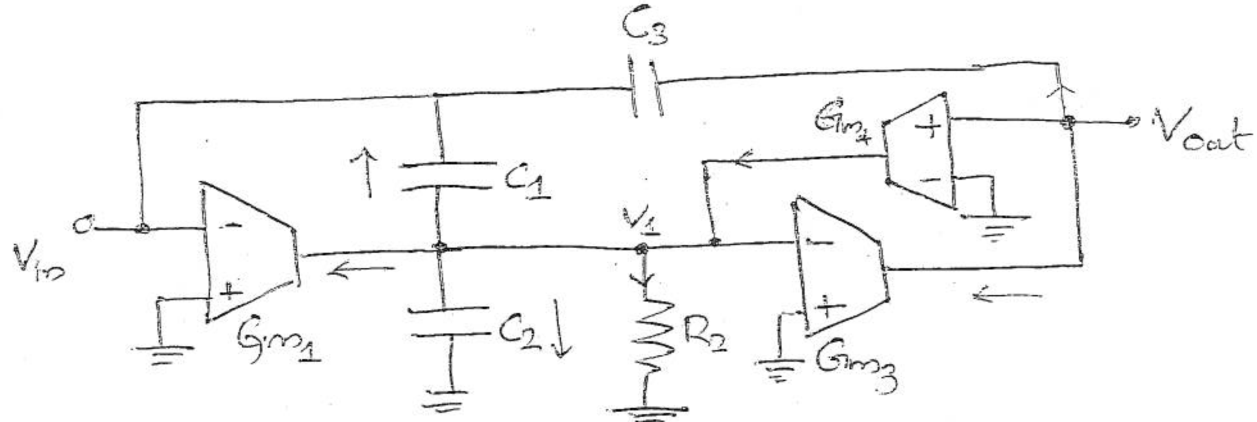
$$\frac{\omega_o}{Q} = \frac{R_1+R_2}{C_2\cdot R_1 R_2}$$

$$Q = \omega_o \times \frac{R_1R_2\cdot C_2}{R_1+R_2}$$

$$= \frac{1}{\cancel{\sqrt{C_2}}\cdot\sqrt{C_1}\,\cancel{\sqrt{R_1R_2}}} \times \frac{\overset{\sqrt{C_2}}{\cancel{C_2}}\cdot\overset{\sqrt{R_1R_2}}{\cancel{R_1R_2}}}{R_1+R_2}$$

$$= \frac{\sqrt{C_2R_1R_2}}{\sqrt{C_1}\cdot R_1+R_2}$$

2.

a) 

Writing nodal equation at V_1

$$\frac{V_1}{R_2} - G_{m_4}V_{out} + (V_1 - V_{in})sC_1 + V_1\, sC_2 + G_{m_1}V_{in} = 0$$

$$V_1\left[\frac{1}{R_2} + sC_1 + sC_2\right] + V_{in}\left[G_{m_1} - sC_1\right] = G_{m_4}V_{out} \quad \text{—— (1)}$$

Writing nodal equation at V_{out}

$$G_{m_3}V_1 + (V_{out} - V_{in})sC_3 = 0$$

$$\Rightarrow \quad G_{m_3}V_1 = (V_{in} - V_{out})sC_3$$

$$\Rightarrow \quad V_1 = (V_{in} - V_{out})\frac{sC_3}{G_{m_3}} \quad \text{—— (2)}$$

Putting (2) in (1),

$$(V_{in} - V_{out})\frac{sC_3}{G_{m_3}}\left[\frac{1}{R_2} + sC_1 + sC_2\right] + V_{in}\left[G_{m_1} - sC_1\right] = G_{m_4}V_{out}$$

Multiplying everything by $R_2 G_{m_3}$

$$\left[V_{in} - V_{out}\right]sC_3\left[1 + s(C_1 + C_2)R_2\right] + V_{in}R_2G_{m_3}\left[G_{m_1} - sC_1\right]$$

$$= R_2G_{m_3} \cdot G_{m_4}V_{out}.$$

$$V_{in}\left[sC_3G_{m_3} + s^2C_3(C_1+C_2)R_2G_{m_3} + R_2G_{m_3}G_{m_1} - R_2G_{m_3}sC_1\right]$$

$$= V_{out}\left[R_2G_{m_3}G_{m_4} + sC_3G_{m_3} + s^2C_3(C_1+C_2)R_2G_{m_3}\right]$$

$$V_{in}\left[s^2(C_1+C_2)C_3\cdot R_2G_{m_3} + s\left[C_3G_{m_3} - R_2G_{m_3}C_1\right.\right.$$

$$(V_{in}-V_{out})\,sC_3\left[1+s(C_1+C_2)R_2\right] + V_{in}R_2G_{m_3}\left[G_{m_1}-sC_1\right]$$

$$= G_{m_4}\cdot R_2G_{m_3}\cdot V_{out}.$$

$$V_{in}\left[sC_3 + s^2C_3(C_1+C_2)R_2\right] + V_{in}\left[R_2G_{m_3}G_{m_1} - R_2G_{m_3}sC_1\right]$$

$$= V_{out}\left[sC_3 + s^2(C_1+C_2)C_3\cdot R_2 + G_{m_4}\cdot R_2G_{m_3}\right]$$

$$V_{in}\left[s^2C_3(C_1+C_2)R_2 + sC_3 - R_2G_{m_3}sC_1 + R_2G_{m_3}G_{m_1}\right]$$

$$= V_{out}\left[s^2C_3(C_1+C_2)R_2 + sC_3 + R_2\cdot G_{m_3}G_{m_4}\right]$$

$$\frac{V_{out}}{V_{in}} = \frac{s^2C_3(C_1+C_2)R_2 + s\left[C_3 - R_2G_{m_3}C_1\right] + R_2G_{m_3}G_{m_1}}{s^2C_3(C_1+C_2)R_2 + sC_3 + R_2G_{m_3}G_{m_4}}$$

$$Q = \frac{\sqrt{C_2 R_1}}{\sqrt{C_1}} \cdot \frac{\sqrt{R_2}}{R_1+R_2}$$

Now to maximise Q

$$\frac{dQ}{dR_2} = 0 \Rightarrow \frac{(R_1+R_2) \times \frac{1}{2\sqrt{R_2}} - \sqrt{R_2} \cdot 1}{(R_1+R_2)^2} = 0$$

$$\Rightarrow \frac{R_1+R_2}{2\sqrt{R_2}} = \sqrt{R_2}$$

$$\Rightarrow R_1 + R_2 = 2R_2$$

$$\Rightarrow \boxed{R_1 = R_2 = 25\ k\Omega}$$

$$Q_{max} = \frac{\sqrt{C_2 R_1 R_2}}{\sqrt{C_1} \cdot R_1+R_2} = \frac{\sqrt{C_2} \cdot R}{\sqrt{C_1} \cdot 2R} = 4$$

$$\sqrt{C_1} = \frac{\sqrt{C_2}}{8}$$

$$\Rightarrow \boxed{C_1 = \frac{C_2}{64} = 0.78125\ pF}$$

Dividing by $C_3(C_1+C_2)R_2$

$$\frac{V_{out}}{V_{in}} = \frac{s^2 + s\,\dfrac{C_3 - R_2 G_{m3} C_1}{C_3(C_1+C_2)R_2} + \dfrac{G_{m1}G_{m3}}{C_3(C_1+C_2)}}{s^2 + \dfrac{s}{(C_1+C_2)R_2} + \dfrac{G_{m3}G_{m4}}{C_3(C_1+C_2)}}$$

a) Inorder to have zeroes located on imaginary axis.

coefficient of 's' term $=0$

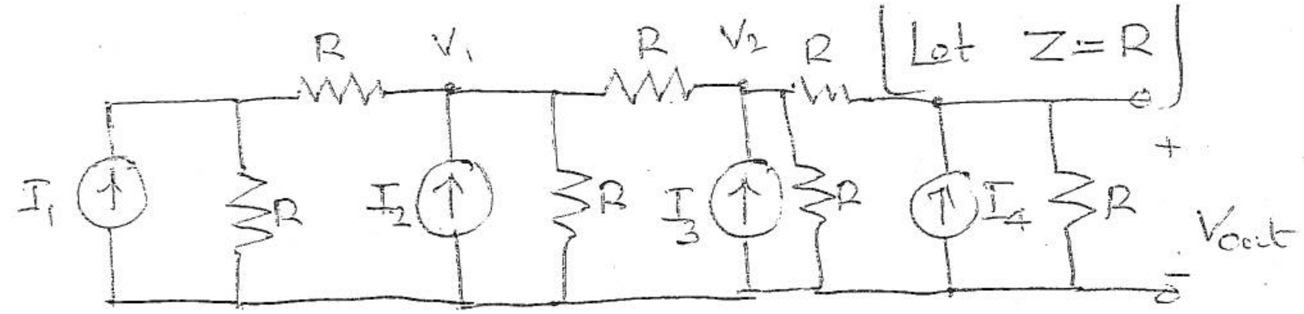
$$C_3 = R_2 G_{m3} C_1$$

$$\Rightarrow \boxed{\frac{C_3}{C_1} = G_{m3} R_2}$$

b) Pole frequency $\omega_0 = \sqrt{\dfrac{G_{m3}G_{m4}}{C_3(C_1+C_2)}}$

$$\frac{\omega_0}{Q} = \frac{1}{(C_1+C_2)R_2}$$

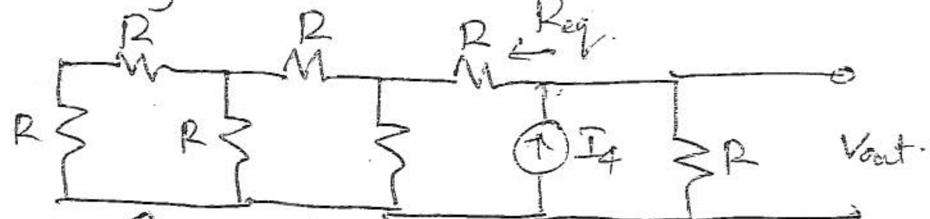
$$\boxed{Q = \sqrt{(C_1+C_2)}\,R_2 \cdot \frac{\sqrt{G_{m3}G_{m4}}}{\sqrt{C_3}}}$$

3) 

[Let Z = R]

Direct Analysis [Super position]

Assuming only I_4 is there



$$R_{eq} = \left\{\left[\left(2R // R\right) + R\right] // R\right\} + R$$

$$= \left[\left(\frac{2R}{3} + R\right) // R\right] + R$$

$$= \frac{5}{8}R + R$$

$$= \frac{13R}{8}$$

$$\frac{2R \times R}{3R}$$

$$\frac{\frac{5R}{3} \cdot R}{\frac{5R}{3} + R}$$

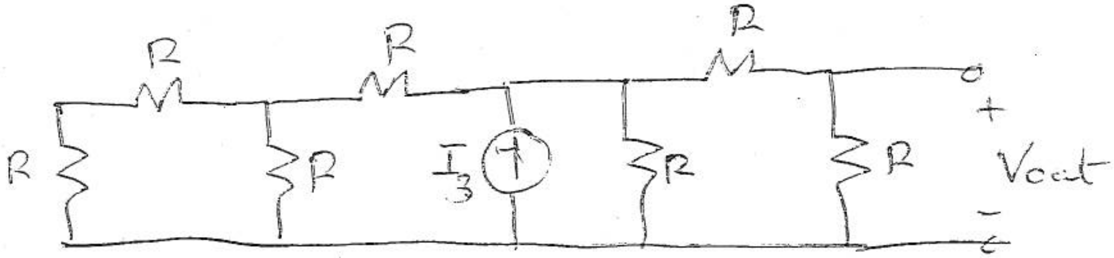
$$\frac{5\cancel{R^2}/3}{8\cancel{R}^2/3}$$

$$\frac{V_{out}}{I_4} = \frac{13R}{8} // R$$

$$= \frac{\frac{13R^2}{8}}{\frac{21R}{8}} = \frac{13R}{21}$$

$$\boxed{\frac{V_o}{I_4} = \frac{13R}{21}}$$

$\|^{ly}$ Only I_3.



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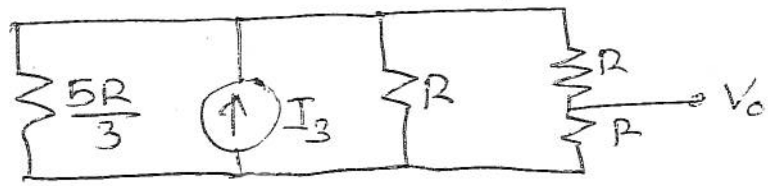

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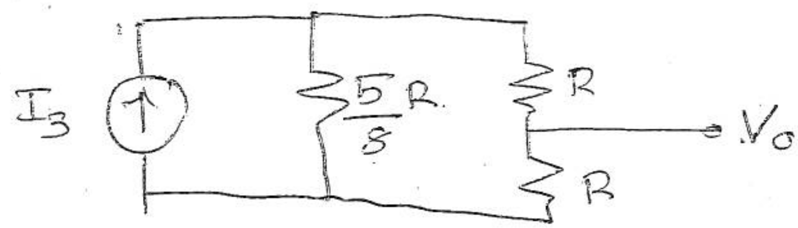
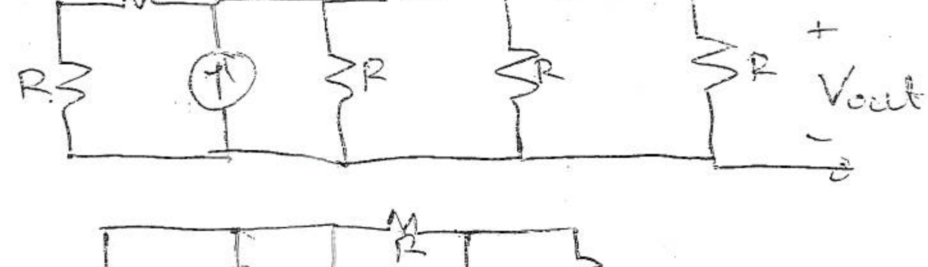
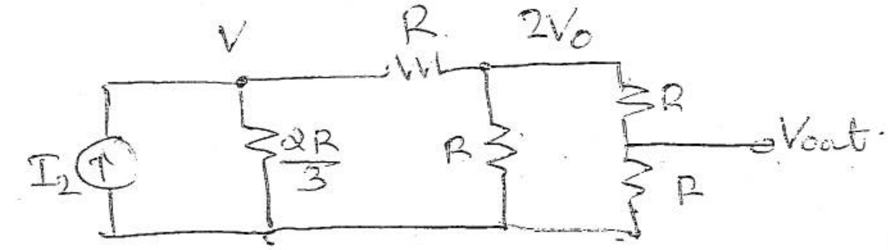


$$V_o = I_3 \times \frac{\frac{5}{8}R}{\frac{5}{8}R + 2R} \times R$$

$$\frac{V_o}{I_3} = \frac{5/\cancel{8}\, R}{\frac{21R}{\cancel{8}}} \times R \qquad \boxed{\frac{V_o}{I_3} = \frac{5}{21}R}$$

$\|^{ly}$ only I_2.





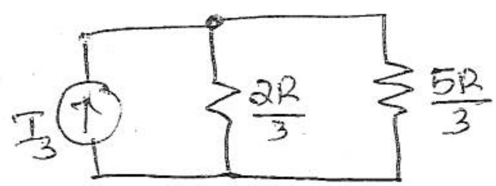
$R + \frac{2R}{3}$

$$\frac{\frac{2R}{3}\times\frac{5R}{3}}{\frac{7R}{3}}$$

$$\frac{10R^2}{\cancel{9}\,3}\times\frac{\cancel{3}}{7R}$$

This can be solved as

$V =$



$$V = I_3 \times \frac{10}{21} R$$

Now $\cancel{\frac{V-2V_o}{R}}$ $\quad \frac{2V_o}{R} + \frac{2V_o}{2R} + \frac{2V_o - V}{R} = 0$

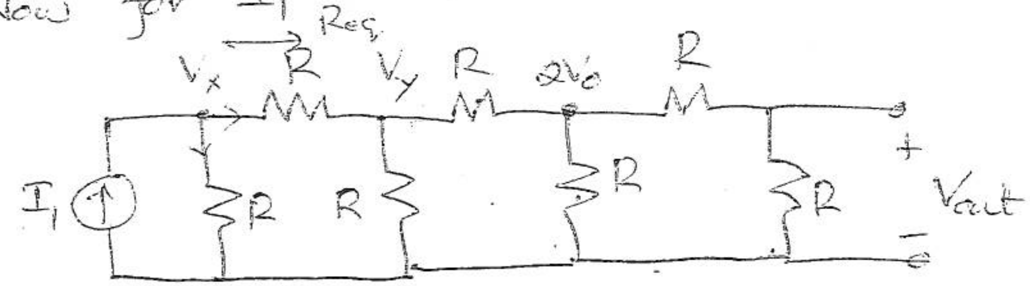
$$2V_o\left[\frac{1}{R} + \frac{1}{R} + \frac{1}{2R}\right] = \frac{V}{R}$$

$$\cancel{2}V_o\left[\frac{5}{\cancel{2}\cancel{R}}\right] = \frac{V}{\cancel{R}}$$

$$V_o = \frac{V}{5} = \frac{I_3}{21} \times \frac{10R}{5}$$

$$\Rightarrow \boxed{\frac{V_o}{I_3} = \frac{2R}{21}}$$

Now for I_1



$$\frac{V_x}{R} + \frac{V_x - V_y}{R} = I_1$$

$$V_x\left[\frac{2}{R}\right] - \frac{V_y}{R} = I_1 \quad \text{——— (i)}$$

$$\frac{V_y - V_x}{R} + \frac{V_y - 2V_o}{R} + \frac{V_y}{R} = 0.$$

~~V_x can be found as~~

~~$I_1 \times R_{eq} // R$~~

~~$R_{eq} = \frac{11R}{3}$~~

~~$V_x = I_1 \times \frac{14R}{3.}$~~

~~Now $V_y = V_x - \left[I_1 - \frac{V_x}{R}\right] R.$~~

~~$= V_x - I_1 R + V_x$~~

~~$= 2V_x - I_1 R$~~

~~$= \left[\frac{28R}{3} - R\right] I_1$~~

$V_y = I_1 \times \frac{25R}{3}$

$\frac{5R}{3} // R$

$\frac{8R}{3} \times R$

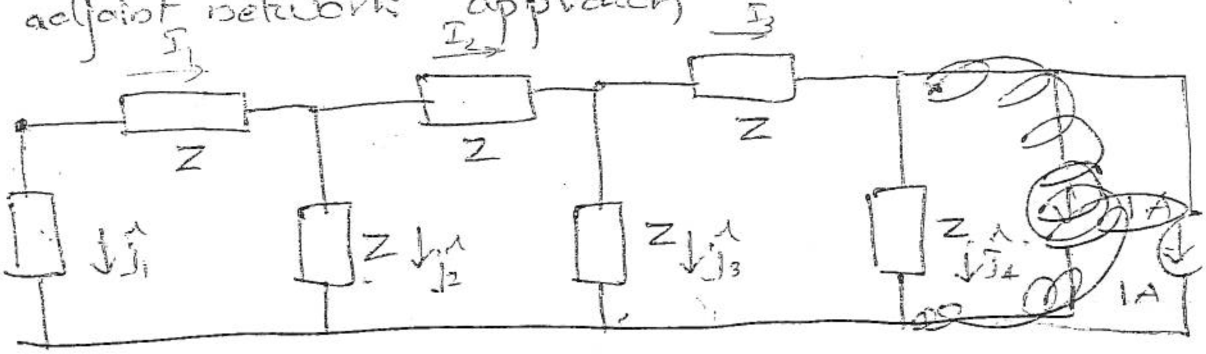
$\frac{2R}{3} // R$

$\frac{2R}{3} + R$

$\frac{5R}{3} // R$

$5R$

Trying adjoint network approach



$$\cancel{\hat{j}_1 Z - \left[\hat{j}_1 Z - \hat{j}_2 Z\right]}$$

$$\frac{5Z}{8}+1$$

$$\hat{j}_1 = +\hat{j}_2 \cdot \frac{Z}{2Z}$$

$$2\times\frac{5}{2}$$

$$\hat{j}_1 = \frac{\hat{j}_2}{2} \quad \text{——} \quad (1)$$

$$5Z/3$$

$$\frac{\hat{j}_3 \cdot Z}{Z} \times \frac{2Z/3}{\frac{2Z}{3}+Z} = \hat{j}_2$$

$$\hat{j}_3 = \hat{j}_2 \times \frac{5}{2} \quad \text{——} \quad (2)$$

$$\frac{\hat{j}_4 \cdot Z}{Z} \times \frac{5Z/8}{\frac{5Z}{8}+Z} = \hat{j}_3$$

$$\Rightarrow \hat{j}_4 = \hat{j}_3 \times \frac{13}{5}$$

$$\hat{j}_4 = \frac{1A}{\cancel{}} \times \frac{13\,\cancel{2}\,Z}{21}$$

$$\Rightarrow \quad \hat{j}_4 = \frac{13}{21} Z$$

$$\hat{j}_3 = \hat{j}_4 \times \frac{\cancel{13}\,5}{13} = \frac{5Z}{\cancel{13}\,21}$$

$$\hat{j}_2 = \hat{j}_3 \times \frac{2}{5} = \frac{2Z}{21}$$

$$\hat{j}_1 = \hat{j}_2 \times \frac{1}{2} = \frac{Z}{21}$$

From interreciprocity and adjoint network analysis

$$\frac{\partial V_o}{\partial I_1} = \hat{j}_1 = \frac{Z}{21}$$

$$\frac{\partial V_o}{\partial I_2} = \hat{j}_2 = \frac{2Z}{21}$$

$$\frac{\partial V_o}{\partial I_3} = \hat{j}_3 = \frac{5Z}{21}$$

$$\frac{\partial V_o}{\partial I_4} = \hat{j}_4 = \frac{13Z}{21}$$

This was verified using direct analysis